Abstract—This paper presents two closed-form localization algorithms, a general algorithm and a co-located algorithm, for distributed multiple-input multiple-output (MIMO) radar systems. In distributed MIMO radar systems, range sum measurements are used to estimate the location parameter. For this, the range sum error minimization is actually employed to be degenerated into two cases for time-of-arrival (TOA) passive localization, one by employing the distance estimate between the target and the receiver (the general algorithm) and the other by subtracting the distance measurement between the target and the transmitter after the time-delay estimation (the co-located algorithm). The resulting positioning accuracy of the general and co-located techniques is found to perform better than that of the existing closed-form weighted least squares (WLS) algorithm and attain the Cramér-Rao lower bound (CRLB).

Index Terms—Best linear unbiased estimator, Cramér–Rao lower bound (CRLB), multiple-input multiple-output (MIMO), position estimation, time delay.

I. INTRODUCTION

The aim of localization is to find a geometrical point of intersection using measurements from each receiver, based on the time difference of arrival (TDOA), the time of arrival (TOA), or the angle of arrival (AOA). The point source localization has been a popular research issue in the radar, sonar, global positioning system, and telecommunication fields [1]-[5]. Recently, intensive research on multiple-input multiple-output (MIMO) systems has been performed on wireless communications and radar systems [6]-[15]. The MIMO radar concept was introduced and significant accuracy improvement in directional finding was provided in [6]. An optimal Neyman-Pearson detector was developed and analyzed for statistical MIMO radar [7]. Waveform diversity in the MIMO radar also has been shown to improve parameter identifiability [8], [9]. In particular, distributed MIMO radar systems assuming homogeneous clutter provided significant performance gains compared with those of phased array radars [10] and these results were extended to the non-homogeneous clutter environments [11]. The maximum likelihood (ML)-based velocity estimation method and the Cramér-Rao lower bound (CRLB) for the velocity estimation were proposed in [12]. The advantages of the MIMO systems were extensively investigated in the wireless communications circumstances [13]-[15]. Source localization is performed for wideband signals using the acoustic sensor arrays where the sources are corrupted by spatially-non-white noise and the expectation maximization (EM) algorithm and direction-of-arrival (DOA) estimation based localization method are utilized [16]. The path delay and DOAs were estimated for a known signal embedded in white Gaussian noise using the antenna arrays [17]. A joint TOA/DOA estimator was developed for ultra-wideband indoor ranging under line-of-sight (LOS) conditions by employing an array of antennas [18]. The time delays and DOAs were determined by utilizing the MIMO arrays and the position of the source was estimated using these information [19]. The bistatic radar sensor networks were used to detect the potential attacks at some points of interest [20] and MIMO radar sensor networks were utilized to solve the multi-target localization problem [21].

In addition, the ultra-wideband (UWB) wireless radar sensor networks (WRSN) were adopted to classify the mobile targets [22]. The time varying wireless channel modeling between MIMO transmitters and receivers, where the pedestrians exist, using the radar cross section modeling was studied [23]. Also, the modeling of the fading channels in distributed radar sensor networks, where the generalized $K$-distribution is approximated by a Gamma probability density function (PDF) using the moment matching method, was proposed in [24]. In particular, the following positioning algorithms for the MIMO radar systems have been proposed. In [25], the ML estimator and CRLB for target direction estimation were derived for an arbitrary signal coherence matrix. The ML estimator of the target direction of departures and DOAs of multiple targets for bistatic MIMO radar systems was discussed [26] and the DOA estimation method using the conjugate ESPRIT for monostatic MIMO radar was proposed in [27]. In addition, the ML position estimate based on coherent processing was obtained [28] and the ML-based method for estimating the parameters of a moving target in non-coherent MIMO radar systems was investigated [29]. The direct position determination algorithm was developed under the assumptions of known and unknown signal waveforms [30]-[32]. A new multiple-hypothesis (MH)-based localization method was suggested in [33]. It is worthwhile to note that the closed-form best linear unbiased estimator (BLUE) was proposed in [34], [35], where the BLUE can be adopted for various signal models if the first and second moments are known [36].

In general, there are two categories of sensor configurations: co-located and widely dispersed radar networks. The direction

Chee-Hyun Park and Joon-Hyuk Chang are with the Department of Electronic Engineering in Hanyang University, Seoul, 133-791, Korea (e-mail: jchang@hanyang.ac.kr).
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2015.2490677, IEEE Transactions on Wireless Communications

Information of the target is found in the former case and the Cartesian coordinates are obtained in the latter [37], [38]. The main focus of this study is the latter case which concentrates on distributed MIMO radar networks. Approaches to the position estimation problem can also be divided into two types: direct and indirect methods. The direct method seeks the location parameter directly from the signal model using the exact ML or the grid search method, whereas the indirect method performs positioning using the TOA measurements. Each method has its own merits. The direct position determination has better accuracy than the indirect method in low SNR regimes and does not require an association step for multi-target situations, but its computational complexity is higher than that of the closed-form algorithm and the initial point must be properly determined.

In contrast, the closed-form solution can be obtained using the indirect localization method, although it demands time-delay estimation before the position is determined and the association procedure for the multi-target problem is required. Actually, the localization performances of two methods are nearly the same as the optimal performance bound in high SNR regimes. An example of the indirect closed-form localization method is the two-step weighted least squares (WLS) method, which has been widely used in wireless communications environments; its localization accuracy approximates that of the CRLB under a sufficiently low noise condition [2], [3]. Furthermore, the root mean squared error (RMSE) performance of the two-step WLS method in low SNR regimes was improved by employing the bias-variance tradeoff property and the shrinkage estimator in [4]. The motivation behind this paper is that the existing localization methods for distributed MIMO radar networks are usually based on the direct localization technique [30]-[32], but they have drawbacks, making the computational complexity higher than that of the closed-form indirect method. This is because they are implemented using iteration-based or grid-search methods, which require the proper initial solution to be known prior for adopting the iteration-based method. The closed-form MIMO radar positioning method discussed in [34], [35] has the disadvantage that it calls for initial target location; thus, it may be impractical in some real circumstances.

We introduce two closed-form localization algorithms for distributed MIMO radar systems in which the a priori location parameter is not needed and the estimation error covariance information of the range estimate between the receivers and target is used. The former is the general algorithm in which the transmitters and receivers can be co-located or non-colocated and the latter is the co-located algorithm in which the transmit and receive element must be at the same site, that is fully netted monostatic radar. In distributed MIMO radar systems, range sum measurements are utilized, whereas range measurements are used for passive TOA localization in wireless communications environments. To convert the range sum error minimization problem in distributed MIMO radar systems into a range error minimization issue for TOA source localization, we adopt the range estimate obtained from the conventional closed-form WLS method [39]. Then, the localization problem which uses the range sum measurement is condensed to the passive localization in which the range estimates obtained from the WLS algorithm are utilized. Subsequently, the TOA passive localization algorithm can be adopted using this distance estimate and the estimation error covariance matrix of the range estimate between receive antennas and target. Note that this estimation error covariance matrix is different from the measurement error covariance matrix directly attained from the range measurement. On the other hand, the co-located algorithm does not require the extra step of finding the range estimates because it can attain these estimates directly from the measurement equation after the time-delay estimation. The simulation results show that the proposed general and co-located algorithms are superior to the closed-form WLS algorithm [39] and approximate the CRLB in the non-collocated transmit/receive sensors configuration and the co-located radar arrangement.

The rest of this paper is organized as follows. Section II explains the problem formulation to be solved in this paper. Section III details the proposed localization methods using time delay measurements. Section IV evaluates the estimation performances of the proposed methods via the simulation results, comparing them with the existing closed-form WLS algorithm and CRLB. Finally, Section V presents the conclusion.

II. PROBLEM FORMULATION

Since the main goal of the time-delay based target localization method is to find the position of a target accurately, the sum of the distance between the ith transmitter and the target and the distance between the target and the jth receiver must be obtained for the localization in distributed MIMO radar networks. Thus, the measurement equation is firstly represented with an added noise component as follows:

\[
 r_{ij} = d_{ij} + n_{ij} = d_{ij} + d_{i} - d_{j} + n_{ij} = \sqrt{(x_{ij} - x)^2 + (y_{ij} - y)^2} + \sqrt{(x_{ij} - x)^2 + (y_{ij} - y)^2} + n_{ij} \tag{1}
\]

where

\[
 n_{ij} \sim N(0, \sigma_{n_{ij}}^2), \quad i = 1, 2, \ldots, M; \quad j = 1, 2, \ldots, N
\]

with M and N denoting the number of transmitters and the number of receivers, respectively. Given M transmitters and N receivers, there are M times N range sum measurements. Also, \( r_{ij} \) is a measurement that is equal to the sum of \( d_{ij} \) (the distance between the target and ith transmitter) and \( d_{ij} \) (the distance between the target and jth receiver), with additive Gaussian noise. We assume that the noise components of the range sum measurements are independent. Note that

\[
 d_{ij} = \sqrt{(x_{ij} - x)^2 + (y_{ij} - y)^2} \quad \text{is the distance between the target and the ith transmitter,}
\]

\[
 d_{ij} = \sqrt{(x_{ij} - x)^2 + (y_{ij} - y)^2} \quad \text{is the range between the target and the jth receiver, [x y]T is the true position of the target, [x_{ij} y_{ij}]T is the position of the ith transmitter and [x_{ij} y_{ij}]T is the coordinates of the jth receiver. Throughout this paper, a lowercase boldface letter denotes a vector, an uppercase boldface letter indicates a matrix and the superscript T signifies the vector/matrix transpose. The}
purpose of this paper is to find the target position for which the RMSE of the position estimate is minimized.

III. PROPOSED CLOSED-FORM LOCALIZATION METHOD

The closed-form time-delay based localization method using a Taylor-series proposed in [34], [35] suffers from the drawback that the initial target coordinates must be known. Also, the algebraic closed-form WLS solution in [39] does not attain the CRLB with the extensive Monte-Carlo simulation environments. To address these problems, in this section, we propose two closed-from localization methods that do not require an initial location estimate and achieve the CRLB. In the existing closed-form WLS algorithm [39], the range between the target and receive elements can be estimated. These range estimates between the target and receivers are utilized in the second-step of the proposed method and the position of the target is found using the closed-form WLS method, in which the estimation error covariance matrix of range estimates is adopted as the weighting matrix. In the third-step, the estimates of the second-step are improved by using the estimation error distribution of the second-step solution. On the other hand, in the co-located algorithm, the range measurement between the transmit/receive pair and the target is subtracted from the range sum measurement. As a result, both localization methods, by using the range sum, boil down to conventional TOA passive source localization algorithms that use the range information between the target and the receiver. This fact indicates that the location parameter is determined as the point of intersection of circles whose radii are respectively equal to the target-receiver range estimate in the general algorithm and the target-receive range measurement in the co-located method. The exact intersection point does not exist due to the measurement noise; thus, the position is estimated as the point that minimizes the sum of squared error.

3.1. General Algorithm for an Arbitrarily Distributed Transmitter and Receiver Configuration

In this section, we propose a localization algorithm in which the transmitters and receivers can be placed in arbitrary position, namely, the transmitters and receivers may be located at the same site or widely separated (see Fig. 1). Fig. 1 shows the two MIMO radar system layouts adopted in this paper. Fig. 1(a) presents the non-colocated configuration in which the transmitters and receivers are spatially separated. The solid line denotes the signal sent by transmitter 1, and the dashed line denotes the signal from transmitter 2. The receive antennas obtain the signals from transmitters 1 and 2. In this formulation, the distances between the receive sensors and the target are unknown, so they must be estimated from the range sum measurements. Fig. 1(b) depicts the arrangement of the radar network with the co-located transmit/receive antennas. Every transmit/receive sensor has four signals from all the transmitters because they are netted (connected). Note that the distances between the target and transmit/receive elements are obtained as \( \{ \frac{x_t}{r_t}, \frac{y_t}{r_t}, \frac{x_r}{r_t}, \frac{y_r}{r_t} \} \) from the measurement equations of the monostatic transmit/receive pairs, unlike the non-colocated formulation where the distances between the target and the receivers must be estimated. By using the range sum measurements shown in Fig. 1(a), the general algorithm is derived in the following. The general algorithm is composed of three steps. The first-step solution is the same as the closed-form WLS estimate [39] and it is obtained in the following: (1) can be reformulated as follows:

\[
 r_{ij} - d_{ij} = d_{ij} + n_{ij}, \quad i = 1, \cdots, M, \quad j = 1, \cdots, N. \tag{2}
\]

Squaring (2) for the \( j \)th receiver yields the following \( M \) equations:

\[
 r_{ij}^2 + x_{ij}^2 + y_{ij}^2 = 2(x_{ij} - x_t)x + 2(y_{ij} - y_t)y + 2r_{ij}d_{ij} + n_{ij}^2 + 2d_{ij}n_{ij}, \quad i = 1, \cdots, M. \tag{3}
\]
Letting $x_1 = [x\ y\ d_{t,1} \cdots d_{t,N}]^T$ and representing (3) in a matrix form yields:

$$b_{i,j} = A_s x_{i} + q_{i,j}$$

(4)

where $b_{i,j} = [b_{i,1}, b_{i,2}, \cdots, b_{i,N}]^T$, $b_{i,j} = \frac{x_i^2 - x_j^2 + y_i^2 - y_j^2}{2}$, $\Delta_{i,j} \in \mathbb{R}^{M \times N}$ has zero entries except $[\Delta_{i,1,j}] = [r_{i,1} \ r_{i,2} \cdots \ r_{i,j}]^T$, $q_{i,j} = [q_{i,1,j}, q_{i,2,j}, \cdots, q_{i,N,j}]^T$, $q_{i,j} = \frac{b_{i,j} - d_{i,j} n_{i,j}}{2}$, $\Delta_x = [a_{r,1} \cdots a_{r,M}]^T$ and $a_{r,i} = [x_{i} - x_{j}, y_{i} - y_{j}]$. Combining each sub-matrix of the $j$th receiver for all $N$ receivers results in

$$b_i = A_s x_i + q_i$$

(5)

where $b_i = [b_{i,1}^T, b_{i,2}^T, \cdots, b_{i,N}^T]^T$, $q_i = [q_{i,1}^T, q_{i,2}^T, \cdots, q_{i,N}^T]^T$ and $A_i = \begin{pmatrix} A_s & \Delta_1 \\ A_s & \Delta_2 \\ \vdots & \vdots \\ A_s & \Delta_N \end{pmatrix}$. The first-step BLUE minimizing the sum of squared error is obtained as follows:

$$\hat{x}_i = (A_i^T C_i^{-1} A_i)^{-1} A_i^T C_i^{-1} b_i$$

(6)

where

$$C_i = \text{cov}(q_i) = \begin{pmatrix} C_{i,1} & 0 & \cdots & 0 \\ 0 & C_{i,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{i,N} \end{pmatrix}$$

(7)

$$C_{i,j} = \text{cov}(q_{i,j}) = \text{diag}[d_{i,j}^2, d_{i,2,j}^2, d_{i,3,j}^2, \cdots, d_{i,M,j}^2, \cdots, d_{i,N,j}^2],$$

$$j = 1, \cdots, N.$$

As can be seen from (7), the error covariance matrix of the WLS estimate (6) requires the range information between the target and transmit elements $(r_{i,j})$ but they are unknown values. Thus, the least squares (LS) estimate is adopted to estimate the target position to determine the distance information between the target and transmit antennas.

Then, the localization problem in which the range sum measurements are used is converted into the TOA passive source localization formulation by utilizing the estimate $\hat{d}_{i,j}$ which is found in (6). The TOA equations from $N$ receivers are represented as below where using these distance estimates:

$$(x - x_{j})^2 + (y - y_{j})^2 = \hat{d}_{i,j}^2, \quad j = 1, \cdots, N$$

$$\approx x^2 + y^2 + x_j^2 + y_j^2 - 2x \cdot x_j - 2y \cdot y_j = \hat{d}_{i,j}^2$$

(8)

$$\Rightarrow x^2 + y^2 + x_j^2 + y_j^2 - 2x \cdot x_j - 2y \cdot y_j = (d_{i,j} + \hat{n}_{i,j})^2$$

$$\Rightarrow x^2 + y^2 + x_j^2 + y_j^2 - 2x \cdot x_j - 2y \cdot y_j = d_{i,j}^2 + \hat{n}_{i,j}^2 + 2d_{i,j} \hat{n}_{i,j}.$$  

Arranging (8) for the common variables and then representing it in a matrix form, we obtain:

$$f_2 = B_2 x_2 + w_2$$

(9)

where $f_2 = [f_1 \cdots f_N]^T$, $f_j = \frac{x_i^2 + y_i^2 - x_j^2 - y_j^2}{2}$, $\hat{d}_{i,j} = d_{i,j} + \hat{n}_{i,j}$, $\hat{x}_i = [\hat{x}_{i,1} \cdots \hat{x}_{i,N}]$, $\hat{n}_{i,j} = [\hat{n}_{i,j}]$, $\hat{x}_i = x_i - x_1 = H_{i,1} q_i$, $H_i = (A_i^T C_i^{-1} A_i)^{-1} A_i^T C_i^{-1}$, $q_i$ is the same as that defined in (4) and (5). $[\Delta_{i,1,j}]$ denotes the $(j + 2)$th component of $\Delta x, [w_2] = [w_{1,2}, \cdots, w_{N,2}]^T$, $v_j = [x_j, y_j - 0.5]$ and $x_2 = [x\ y\ R]^T$ and $R = x^2 + y^2$. Subsequently, the second-step location estimator is found:

$$\hat{x}_2 = (B_2^T C_2^{-1} B_2)^{-1} B_2^T C_2^{-1} f_2$$

(10)

where $C_2 = \text{cov}(w_2)$. The estimation error covariance matrix $\text{cov}(w_2)$ is equal to

$$[C_{2,j,k}] = E\{w_{j,i} w_{k,i}\} = d_{i,j} t_{j,k} E\{n_{i,j} n_{k,j}\}$$

$$\approx \frac{t_{j,k}}{t_{r,1,k} E\{[\Delta_{i,1,j}][\Delta_{i,1,k}]\}}$$

$$l = 1, \cdots , N, \quad k = 1, \cdots , N$$

(11)

where $w_j$ denotes the $h$th component of $w_2$, $d_{i,j}$ is the true range among the $h$th receiver and target, and $t_{j,k}$ is the range measurement between the $h$th receive element and target and $\hat{n}_{i,j}$, $\hat{x}_i$ were defined in (9). Then, the estimation error covariance matrix (11) is represented as follows:

$$[C_{2,j,k}] = E\{w_{j,1} w_{k,1}\}$$

$$\approx \frac{t_{j,k}}{t_{r,1,k} E\{[\Delta_{i,1,j}][\Delta_{i,1,k}]\}}$$

$$l = 1, \cdots , N, \quad k = 1, \cdots , N$$

where $([A_1^T C_1^{-1} A_1]^{-1})_{m,n}$ denotes the $(m, n)$th element of the matrix $(A_1^T C_1^{-1} A_1)$. The likelihood function for $f_2$ can be regarded as the multivariate normal distribution by assuming sufficiently small noise conditions, then $B_2^T C_2^{-1} f_2$ is the sufficient statistic for $x_2$ [40]. Thus, the entire information for $x_2$ which can be obtained from $b_i$ also can be attained from $f_2$ because $f_2$ is also the sufficient statistic for $x_2$. Note that the dimension of statistic is reduced from $MN$ to $N$ by the use of $f_2$. The second-step WLS estimate is further improved using the following relation between the estimates:

$$\hat{h} = G x_3 + z$$

(13)

where

$$\hat{h} = [\hat{x}_{2,1}^T \hat{x}_{2,2}^T \hat{x}_{2,3}]^T$$

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$x_3 = [x^2 \ y^2]^T$$

$$z = [2x(\hat{x}_{2,2} - x) \ 2y(\hat{x}_{2,2} - y) \ \hat{x}_{2,3} - R]^T$$

$$R = x^2 + y^2.$$

As a result, the third-step estimate that minimizes the squared error of (13) is obtained by using the BLUE:

$$\hat{x}_3 = (G^T C_h^{-1} G)^{-1} G^T C_h^{-1} \hat{h}$$

(14)

where $C_h = \text{diag}[2(2x \ 2y \ 1)(B_2^T C_2^{-1} B_2)^{-1}\text{diag}[2x \ 2y \ 1]]$, $[\cdot]_k$ means the $k$th element of $[\cdot]$ and $x$ and $y$ are substituted as $[\hat{x}_{2,1}]_1$ and $[\hat{x}_{1,2}]_2$ in the computation of $C_h$. The covariance matrix $C_h$ approximates the true error covariance matrix by using the
delta method [40], thus its error can be large when \( \hat{x}_2 \) is far from the true value \( x_2 \). The final closed-form location estimate is:

\[
\hat{x}_2 = \left[ \text{sgn}(\hat{x}_{31})\sqrt{|\hat{x}_{31}|} \right] \left[ \text{sgn}(\hat{x}_{32})\sqrt{|\hat{x}_{32}|} \right]^T
\]

(15)

where \( \text{sgn}(\cdot) \) denotes the sign function.

3.2. Localization Algorithm for Co-located Transmit and Receive Elements (Fully Netted Monostatic System)

In the previous section, we dealt with the case in which the transmit and receive antennas can be co-located or non-colocated. In this section, we propose a localization method in which the transmitter and receiver should be co-located; this formulation can be seen as a fully netted monostatic radar system (see Fig. 1(b)). By using this algorithm, we can obtain the range measurements between the target and the co-located transmitter/receiver pairs after the time delay estimation, i.e., \( \left\{ \frac{\lambda}{2T}, \ldots, \frac{N\lambda}{2T} \right\} \). This contrasts with the general method, which must estimate the range using the BLUE. Thus, the number of estimation step is smaller than that of the general algorithm. The co-located positioning algorithm is derived as follows. Again, the localization problem that uses the range sum measurements is condensed to the TOA passive source localization formulation by subtracting \( \frac{\lambda}{2T} \) \( (i = 1, \ldots, M) \) from the range sum measurements \( r_i \). By using (1), the TOA equations from \( N \) receivers for the \( i \)th transmitter are:

\[
(x - x_{ti})^2 + (y - y_{ti})^2 = (d_{ti} - d_{ij})^2, \quad j = 1, \ldots, N
\]

\[
\approx x^2 + y^2 + x_{ti}^2 + y_{ti}^2 - 2x \cdot x_{ti} - 2y \cdot y_{ti} = (r_{ti} - \frac{\lambda}{2T})^2
\]

\[
\Rightarrow x^2 + y^2 + x_{ti}^2 + y_{ti}^2 - 2x \cdot x_{ti} - 2y \cdot y_{ti} = (d_{ti} - d_{ij})^2 - (r_{ti} - \frac{\lambda}{2T})^2
\]

\[
= (d_{ti} - d_{ij} - n_{ij}^2) (d_{ti} - d_{ij} - n_{ij}^2)
\]

\[
\Rightarrow x^2 + y^2 + x_{ti}^2 + y_{ti}^2 - 2x \cdot x_{ti} - 2y \cdot y_{ti} = (d_{ti} - d_{ij} - n_{ij}^2)^2 + (n_{ij}^2 - \frac{\lambda}{2T})^2 + 2(d_{ti} - d_{ij} - n_{ij}^2) n_{ij}^2
\]

(16)

Arranging (16) for the common variables and then putting it into matrix form, we get:

\[
c_{s,i} = B_{s} x_{s,i} + e_{s,i} \quad (17)
\]

where \( c_{s,i} = [f_{s,i} \cdots f_{s,i,N}]^T, f_{s,i} = \frac{x_{ti}^2 + y_{ti}^2 - (r_{ti} - \frac{\lambda}{2T})^2}{2}, e_{s,i} = [e_{s,i,1} \cdots e_{s,i,N}]^T, e_{s,i,1} = ([d_{ti} - d_{ij} - n_{ij}^2] n_{ij}^2 + n_{ij}^2 - \frac{\lambda}{2T}) \approx (d_{ti} - d_{ij} - n_{ij}^2)(n_{ij}^2 - \frac{\lambda}{2T}), B_{s} = [x_{s,1} \cdots x_{s,N}]^T, V_{s,i} = [x_{s,i} y_{s,i} - 0.5], x_{s,i} = [x y R]^T and R = x^2 + y^2. Merging each sub-matrix of the \( i \)th transmitter for all \( M \) transmitters, we obtain

\[
c_1 = B_1 x_1 + e_1
\]

(18)

where \( c_1 = [c_{s,1,1} \cdots c_{s,1,M}]^T, e_1 = [e_{s,1,1} \cdots e_{s,1,M}]^T \) and \( B_2 = \begin{bmatrix} B_1^T & B_1^T & \cdots & B_1^T \end{bmatrix}^T \). Then, the first-step location estimator is

\[
\hat{x}_{1} = (B_1^T B_1)^{-1} B_1^T e_1
\]

(19)

where \( R_1 = \text{cov}[e_1] \) and \( \text{cov}(e_1) \) is equal to

\[
E\{ e_{s,j} | e_{s,j} \}
\]

\[
= \begin{cases} (d_{ti} - d_{ij})^2 \cdot \text{var}(\frac{\lambda}{2T}) \\
(d_{ti} - d_{ij})^2 \cdot \frac{\sigma_i^2}{2}, \quad \text{if } i = l, j \neq p \text{ and } i \neq j \\
(d_{ti} - d_{ij})^2 \cdot \{ -\text{var}(\frac{\lambda}{2T}) \} \quad \text{if } i = l, j \neq p \text{ and } i \neq j; \\
(d_{ti} - d_{ij})^2 \cdot \{ -\text{var}(\frac{\lambda}{2T}) \} \quad \text{if } i = l, j \neq p \text{ and } i = j; \\
(d_{ti} - d_{ij})^2 \cdot \{ -\text{var}(\frac{\lambda}{2T}) \} \quad \text{if } i = l, j \neq p \text{ and } i = j; \\
(d_{ti} - d_{ij})^2 \cdot \frac{\sigma_i^2}{2}, \quad \text{if } i = l, j \neq p \text{ and } i \neq j; \\
0, \quad \text{if } i = l, j \neq p \text{ and } i \neq j; \\
\end{cases}
\]

In the same manner as in (13) and (14), the second-step estimate is obtained by improving the first-step estimate using the relationships between the first-step estimates:

\[
\hat{x}_{2,i} = (G^T C_{h}^{-1} G)^{-1} G^T C_{h}^{-1} \hat{h}_i
\]

(20)

where

\[
\hat{h}_i = \begin{bmatrix} \hat{x}_{i,1}^2 & \hat{x}_{i,1} \hat{x}_{i,2} & \hat{x}_{i,2} \end{bmatrix}^T \]

\[
C_{h} = \text{diag}[2x 2y 1] [B_2^T R^{-1} B_2]^{-1} \text{diag}[2x 2y 1]
\]

\[
G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
\left[ \right]_k \text{ means the } k\text{th element of }[\cdot] \text{, and } x \text{ and } y \text{ are replaced by } \hat{x}_{i,1}, \hat{x}_{i,2} \text{ in the computation of } C_{h}. \text{ The final closed-form position estimate is:}
\]

\[
\hat{x}_{f,i} = \left[ \text{sgn}(\hat{x}_{i,1})\sqrt{\hat{x}_{i,2}} \right] \left[ \text{sgn}(\hat{x}_{i,2})\sqrt{\hat{x}_{i,2}} \right]^T
\]

(22)

The co-located method is similar with that of the general algorithm with regard to the localization accuracy, but inferior with respect to the surveillance range and less robust to the destruction of sensors.
IV. SIMULATION RESULTS

In this section, the RMSE performance of the proposed localization method was compared with the existing closed-form WLS algorithm [39] and CRLB. For setting in the simulation, the target was assumed to be located within a $10 \times 10$ km$^2$ region to determine the performance over the entire area and 30 uniformly distributed target locations were generated. Two hundred Monte-Carlo simulations were performed for each given noise variance. The variances of the noise of all of the range sum measurements were assumed to be identical. The RMSE average was calculated as follows:

$$\text{RMSE average} = \sqrt{\sum_{i=1}^{30} \sum_{k=1}^{200} [(\hat{x}^k(i) - x(i))^2 + (\hat{y}^k(i) - y(i))^2]}\over 30 \times 200$$

(23)

where $\hat{x}^k(i)$, $\hat{y}^k(i)$ is the estimated position of the target in the $i$th position set and $k$th iteration, $x(i)$ and $y(i)$ indicate the $i$th true position of the target. Indeed, Fig. 2 shows the configuration of the transmitters and receivers in which all transmit/receive elements are separated and Fig. 3 illustrates the deployment of the transmit and receive antennas in the co-located radar network. The resulting localization accuracy as a function of the noise variance is shown in Fig. 4 for the con-
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2015.2490677, IEEE Transactions on Wireless Communications

The information matrix (FIM) is derived by using the chain rule. The Fisher information matrix (FIM) is:

\[ \text{FIM}(\mathbf{u}) = \frac{1}{\sigma^2} \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right]^T \left[ \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right] \]

(24)

where \( \mathbf{f} = [\mathbf{f}_1, \ldots, \mathbf{f}_N]^T \), \( \mathbf{f}_{s,i} = [f_{s,1}, \ldots, f_{s,N}]^T \), and

\[ \frac{\partial \mathbf{f}_i}{\partial \mathbf{u}} = \left( \begin{array}{c} \frac{\partial f_{1,i}}{\partial u} \\ \frac{\partial f_{2,i}}{\partial u} \\ \vdots \\ \frac{\partial f_{N,i}}{\partial u} \end{array} \right) \]

\[ \frac{\partial f_{s,i}}{\partial x} = \frac{x - x_{r,i}}{\sqrt{(x - x_{r,i})^2 + (y - y_{r,i})^2}} + \frac{x - x_{t,i}}{\sqrt{(x - x_{t,i})^2 + (y - y_{t,i})^2}} \]

\[ \frac{\partial f_{s,i}}{\partial y} = \frac{y - y_{r,i}}{\sqrt{(x - x_{r,i})^2 + (y - y_{r,i})^2}} + \frac{y - y_{t,i}}{\sqrt{(x - x_{t,i})^2 + (y - y_{t,i})^2}} \]

\[ i = 1, \ldots, M, \quad j = 1, \ldots, N. \]

The CRLB is obtained as follows:

\[ \text{CRLB}(x(i)) = [(\text{FIM}(\mathbf{u}))^{-1}]_{1,1} \]

\[ \text{CRLB}(y(i)) = [(\text{FIM}(\mathbf{u}))^{-1}]_{2,2}. \]

(26)

Indeed, Fig. 5 shows a comparison of the RMSE averages of the general algorithm and the co-located algorithm with the existing closed-form WLS estimator and CRLB average when the sensor deployment shown in Fig. 3(a) and (b) is utilized. The CRLB average was obtained in the following:

\[ \text{CRLB average} = \sqrt{\frac{\sum_{i=1}^{30} \left[ \text{CRLB}(x(i)) + \text{CRLB}(y(i)) \right]}{30}}. \]

(27)

The CRLB average is similar with that of the third-step estimate, it is much degraded compared to that of the third-step estimate in the semi-circular layout. Fig. 6 shows the RMSE average performance of the general algorithm with single and multi-sample case when the variance of noise is much large (threshold region) for the deployment of transmit/receive antenna pairs shown in Fig. 2(a). The number of samples was 10 in the general algorithm when multiple samples were utilized. The RMSE average with single sample scenario began to digress from the CRLB average from 55 dB, on the other hand, that of multi-sample case began to degrade from about 65 dB than the CRLB average. In the multiple samples environment, the average of samples, \( \tilde{r}_{i,j} = \frac{1}{P} \sum_{k=1}^{P} r_{i,j}(k) \) (\( k \): the index for samples, \( P \): the number of samples) was adopted instead of \( r_{i,j} \) in (4) and this can be regarded as the MLE for \( d_{i,j} \).
in multiple samples case by the invariance property of the MLE [40]. The invariance property means that if $\hat{\theta}$ is the ML estimator of $\theta$ and $g(\theta)$ is any transformation of $\theta$, then the ML estimator for $g(\theta)$ is by definition $g(\hat{\theta})$. Also, Fig. 7 shows the RMSE average of the general algorithm as a function of the variances of the position errors of the transmitters and receivers when using the transmitter and receiver layout of Fig. 2(a). It can be seen that the RMSE average of the general algorithm is insensitive to the variances of the position errors of the transmit/receive sensor position errors up to -25 dB. Fig. 8 shows the RMSE average performance of the general and co-located algorithms as a function of the variances of the position errors of the transmit and receive antennas in the placement of Fig. 3(a). The RMSE averages of the general and co-located algorithms are insensitive to the antenna position errors up to -30 dB. The proposed localization algorithms can be extended to the positioning method for which the information for the sensor position errors are required in the straightforward manner. However, the variance of the sensor position error should be known a priori, thus it can be impractical in some adverse environments. It can be seen that in Fig. 9(a), the number of receivers was fixed to 7 and the number of transmitters was increased from 5 to 13. On the contrary, the number of receivers varied and that of the transmitters was constant in Fig. 9(b). In both cases, the RMSE averages decrease as the number of transmit/receive antenna pairs in Fig. 10. Again, the RMSE averages decrease as the
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TWC.2015.2490677, IEEE Transactions on Wireless Communications

Fig. 8. RMSE averages of the general and co-located algorithms as a function of the variance of the antennas position errors in the deployment of Fig. 3(a)

V. CONCLUSIONS

Closed-form localization methods for the distributed MIMO radar systems were developed. The proposed methods convert the range sum error minimization problem into the range error minimization used in the TOA passive positioning formulation. This was achieved by adopting the range estimates between the target and receivers attained from the first-step estimate of the general algorithm. The BLUE was employed utilizing the estimation error covariance matrix of the the first-step range estimates between the target and receivers. The simulation results showed that the accuracy of the general and co-located techniques outperformed that of the existing closed-form WLS estimator and approximated the CRLB in the entire SNR regimes.

VI. APPENDIX

In this appendix, we provide the proof that the mean squared error (MSE) of the general algorithm attains the CRLB as follows. Also, the RMSE can be obtained by taking the square root of the MSE. The estimation error $\Delta \hat{\mathbf{x}}_y$ is represented as

$$
\Delta \hat{\mathbf{x}}_y = D_4^{-1} \Delta \hat{\mathbf{x}}_3 = D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \hat{\mathbf{h}} - \mathbf{Gx}_3)
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\hat{\mathbf{x}}_2 - \mathbf{x}_2))
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{B}_2))^{-1} \mathbf{B}_2^T \mathbf{C}_2^{-1}
$$

$$
\times (\mathbf{f}_2 - \mathbf{B}_2 \mathbf{x}_2)
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{B}_2))^{-1}
$$

$$
\times \mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{D}_2 \mathbf{D}_1 (\hat{\mathbf{x}}_1 - \mathbf{x}_1)
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{B}_2))^{-1}
$$

$$
\times \mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{D}_2 \mathbf{D}_1 (\hat{\mathbf{x}}_1 - \mathbf{x}_1)
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{B}_2))^{-1}
$$

$$
\times \mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{D}_2 \mathbf{D}_1 (\hat{\mathbf{A}}_T^T \hat{\mathbf{C}}_1^{-1} \hat{\mathbf{A}}_1)^{-1} \hat{\mathbf{A}}_T^T \hat{\mathbf{C}}_1^{-1} (\mathbf{b}_1 - \mathbf{A}_1 \mathbf{x}_1))
$$

$$
= D_4^{-1} (G^T \mathbf{C}_h^{-1} \mathbf{G}^{-1} G^T \mathbf{C}_h^{-1} \mathbf{D}_3 (\mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{B}_2))^{-1}
$$

$$
\times \mathbf{B}_2^T \mathbf{C}_2^{-1} \mathbf{D}_2 \mathbf{D}_1 (\hat{\mathbf{A}}_T^T \hat{\mathbf{C}}_1^{-1} \hat{\mathbf{A}}_1)^{-1} \hat{\mathbf{A}}_T^T \hat{\mathbf{C}}_1^{-1} \mathbf{D}_0 \Delta \mathbf{r}_A
$$

(28)
where \( D_0 = \text{diag}(d_{s,1}d_{b,2} \cdots d_{s,N}) \), \( D_{a,j} = \text{diag}(d_{a,1} d_{a,2} \cdots d_{a,M}) \), \( D_2 = \text{diag}(d_{a,1} d_{a,2} \cdots d_{a,N}) \), \( D_3 = \text{diag}(2[2x \times y]) \), \( D_k = 2\text{diag}[x \times y] \), \( \Delta r_A = [n_{T,1}^T n_{T,2}^T \cdots n_{T,N}^T]^T \), \( n_{A,j} = [n_{1,j} n_{2,j} \cdots n_{M,j}]^T \) and \( \Omega_{N \times N} \) is the matrix with \( N \) rows and \( 2 \) columns composed of zeros and \( I_{N \times N} \) is the identity matrix with \( N \) rows and \( N \) columns. Then, the error covariance matrix of \( \tilde{x}_f \) is represented as follows:

\[
\text{cov}([\Delta \tilde{x}_f]) = HD_2 D_1 (A_f^T C_1^{-1} A_1)^{-1} D_1^T D_2 H^T \quad (29)
\]

where \( H = D_4^{-1}(G_1^T C_1^{-1} G_1) \), \( G_1 = D_1 (B_2^T C_2^{-1} B_2 - B_1^T C_1^{-1} B_1)^{-1} B_2^T C_2^{-1} \). The CRLB \((x)\) is obtained as given below:

\[
\text{CRLB}(x) = \left[ (\partial r_A / \partial x)^T Q_{\Delta r_A} (\partial r_A / \partial x) \right]^{-1} \quad (30)
\]

where \( Q_{\Delta r_A} = E([\Delta r_A(\Delta r_A)^T]\) and \( D_0 Q_{\Delta r_A} D_0^T = C_1 \). From the last equation of (27), we obtain the following relationship by Taylor series expansion

\[
(\partial r_A / \partial x) \simeq (HD_2 D_1 (A_f^T C_1^{-1} A_1)^{-1} A_f^T C_1^{-1} D_0)^{-1} \quad (31)
\]

Substituting \((\partial r_A / \partial x)\) into (29) and arranging yield the following CRLB \((x)\):

\[
\text{CRLB}(x) = HD_2 D_1 (A_f^T C_1^{-1} A_1)^{-1} D_1^T D_2 H^T \quad (32)
\]

Thus, the CRLB \((x)\) is identical with the error covariance matrix of \( \tilde{x}_f \) from (28) and (31).

ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (No. 2014R1A2A1A10049735).

REFERENCES


